Techniques for Implementing Structural Model Identification Using Test Data

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Structural system identification methods are analytical techniques for reconciling test data with analytical models. For system identification to become a practical tool for engineering analysis, the estimation techniques/codes must communicate with finite element software packages, without intensive analyst intervention and supervision. This paper presents a technique used to integrate commercial software packages for finite element modeling, mathematical programming techniques, and general linear system analysis. Two examples of application of this software using measured data are presented. The examples consist of a truss structure in which the model form is well defined, and an electronics package whose model form is ill defined since it is difficult to model such structures with finite elements. A comparison of the resulting updated models with the experimental data shows significant improvement.

Nomenclature

$f(\,)$	= nonlinear vector function
$P_{\tilde{x}}$	= covariance matrix of parameter estimate error
P_0	= covariance matrix of initial parameter estimate
R	= covariance matrix of measurement noise
V	= noise vector
W_1, W_2	= objective function weighting matrices
W_x	= parameter weighting matrices
W_z	= measurement weighting matrices
$X(\,)$	= parameter vector
$\hat{X}$	= estimated parameter vector
X^n	= nominal parameter vector
X^0	= initial parameter vector
$\tilde{X}$	= estimated parameter error vector
$Z(\,)$	= measurement vector

I. Introduction

THE development of an adequate mathematical model for a structural system is one of the basic requirements of engineering analysis. The model is used as a tool for design evaluation, test planning, component design specifications, and other analyses. Comparison of the model response with test data is frequently used as a measure of the model's accuracy; however, the results are often less than satisfactory, resulting in the need to modify or update the model. Ad hoc procedures are often used to modify the model; but, for large order, complex finite element models, this method rapidly proves inadequate. Parameter identification is a methodology for systematically and directly updating a mathematical model to achieve better correlation with test data. For system identification to become a practical tool for engineering analysis, the estimation techniques/codes must communicate with finite element software packages, without intensive analyst intervention and supervision.

The current state of the art in system identification for structural dynamics applications generally consists of three basic approaches: 1) one-step algorithms that involve no iterations; 2) iterative methods that are either deterministic or statistically based; and 3) general mathematical programming and optimization techniques. The one-step methods are the least computationally intensive and are based on a closed-form minimization problem. They estimate individual entries in the stiffness and mass matrices; therefore, they do not require any element level finite element model definitions. The resulting updated mass and stiffness matrices exactly reproduce the experimental modal frequencies and mode shapes at a specified number of preselected points on the structure.[1-4] Most of these methods do not directly include provisions to account for uncertainty in the data; however, the more recent advances retain the model connectivity. In addition to model updating, these methods can be used as a preliminary step to locate areas of greatest error in the model; therefore, the results may be used as a guide for selection of physical model parameters to be estimated using other methods.

The general class of iterative methods[5-10] includes nonlinear least-squares-based techniques, which assume a deterministic framework, and Bayesian or Kalman Filter-based techniques, which pose the estimation problem in a statistical framework for both the model parameters and the experimental data. These approaches require design sensitivity information and include both batch and recursive solution procedures. They also permit simple constraints to be imposed on the estimated parameters. These methods require the analyst to select the uncertain parameters a priori, which is sometimes very difficult.

The mathematical programming/optimization techniques have not yet been extensively applied to system identification of dynamic structural models, but have had great success in applications to structural optimization of large finite element models. Optimization methods also employ an iterative approach using design sensitivities, and they permit very general constraint relationships to be imposed among the estimated parameters and dynamic response quantities.[11-13]

One of the difficulties that must be overcome in practical system identification applications is the linking or incorporation of the finite element code and the estimation software. In general, one-step methods avoid this difficulty and require significantly less programming. Recent system identification applications utilizing statistical and optimization techniques have resulted in efforts to produce software that links finite element codes and parameter identification codes.[10-13] Some of these attempts incorporated special-purpose finite element code and estimation code in a single piece of software; however, the software was limited to small sized problems. Other

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applications linked existing finite element codes to estimation software, utilizing either Bayesian, least squares, or optimization algorithms. Such software is written primarily for inhouse use and is typically only semiautomated. There does not currently appear to be any commerically available software for solving the structural system identification problem in a truly automated fashion.

The software written for this study is a portion of a larger effort currently underway at Sandia National Laboratories that will ultimately link a general-purpose FE code with a general-purpose estimation code that will offer all three of the above-mentioned estimation methodologies as options. The work reported in this paper is specialized to two specific problems, but contains the skeleton of the software required for implementing the general software as described above.

This paper discusses the implementation of a tool for system identification of large structural models by integrating commerical software packages. This method integrates the commercial software packages MSC/NASTRAN,[14] PRO-MATLAB,[15] and ADS.[16] The most significant piece of software that needed to be written was NASMAT,[17] which interprets NASTRAN OUTPUT2 tables and matrices for use in PRO-MATLAB as well as other matrix analysis and control design software packages.

Use of the automated parameter identification software is illustrated with the following two applications, which are very different systems:

1) *The estimation of material constants and support stiffnesses of a truss structure*—This problem has a small number of uncertain parameters and the model form is well known and easily modeled using finite element methods. The purpose of using system identification on this problem was to produce an accurate model for use in a structural control experiment.

2) *The estimation of the stiffness and mass properties of an electronics package*—This problem has a large number of uncertain parameters, and the model form is not well known. The development of an appropriate finite element model for this system is not straightforward. The purpose of using system identification on this problem was to produce an accurate model for use in response prediction. The prediction results would be used in design evaluation and component shock and vibration specifications.

In both cases modal test data were used in the estimation procedures. A modal test of each structure was performed and a number of modal frequencies were measured for each structure. The experimental data were then used to update the parameters until the desired level of agreement with the experimental data was obtained.

Two system identification techniques were implemented for this study:

1) *Weighted least squares or Bayesian estimation*, which is most useful for applications containing a few critical, but uncertain parameters. This approach uses covariance matrices to weight the confidence in the initial parameter values as well as the uncertainty in the test data. Statistics are also obtained for the errors in the final parameter estimates.

2) *Mathematical programming techniques*, which are very flexible in the problem statement and have the ability to deal with large numbers of parameters. Parameter values can be constrained to certain regions and general nonlinear constraints can be imposed; however, statistics regarding the error in the final parameter estimates are not currently available with these techniques.

II. Implementation

The system identification techniques illustrated in this paper were implemented by integrating the commercial software packgaes. MSC/NASTRAN was used for the structural modeling, analysis, and sensitivity calculations. ADS is an optimization software package that was used to implement the mathematical programming techniques. PRO-MATLAB is a matrix analysis package, which has capabilities in a variety of areas of linear system analysis. PRO-MATLAB was used to control the flow of the overall system identification problem. ADS was directly linked to PRO-MATLAB; and MSC/NASTRAN communicated with PRO-MATLAB using NASMAT, which is capable of interpreting MSC/NASTRAN matrices and tables. PRO-MATLAB performed the following tasks in the overall system identification procedure:

1) The control of the system identification algorithm was performed.

2) MSC/NASTRAN matrices and tables were acquired using NASMAT.

3) The weighted least squares algorithm was implemented.

4) The objective function and constraint evaluations as required by ADS were performed.

5) MSC/NASTRAN data were edited and the appropriate analysis was performed to recalculate the system response and sensitivities.

The computation time required for system identification using this technique is largely dominated by the execution of the finite element code. The *ideal* system identification software would contain parameter estimation, optimization, and a general-purpose finite element capability within one code; however, an existing commercial software package does not exist with all of these capabilities. To write a software package that is truly general in all aspects and allows user-tailoring of estimation techniques and objectives would be very difficult to produce and maintain. Therefore, the approach of integrating existing commercial software packages, which are general in their individual areas, and writing the unavailable pieces of software, is believed to be the most cost-effective and technically prudent approach.

III. Parameter Estimation Algorithms

The problem of parameter estimation can be described as one of computing a value for an unknown state or parameter vector, $X(k)$ from a given measurement sequence $Z(k)$, $k = 1, 2, \ldots, j$, where a causal relationship between X and Z is assumed:

$$Z(k) = f(X(k)) + V \tag{1}$$

In general, the value of the measurement will be in error due to measurement noise. Linear estimation theory involves those estimation algorithms that permit only linear operations on the measurement vector. One approach to nonlinear estimation problems is to employ successive linearization and apply linear estimation algorithms to the linearized problem. When $f(\cdot)$ is a nonlinear function of the parameters, such as eigenfrequencies, the measurements are related to the parameters via an implicit nonlinear equation, which can be expanded in a Taylor series about a nominal parameter vector, X^n:

$$Z = f(X^n) + \left[\frac{\partial f(X)}{\partial X}\right]_{X=X^n} (X - X^n) + V + \cdots \tag{2}$$

Neglecting the higher order terms the equation may be written

$$\delta Z = A\delta X + V \tag{3}$$

where

$$\delta Z = Z - f(X^n) \tag{4}$$

$$\delta X = X - X^n \tag{5}$$

$$A = \left[\frac{\partial f(X)}{\partial X}\right]_{X^n} \tag{6}$$

This is a linear estimation problem where the measurement variation δZ depends on the parameter variation δX via a linear equation containing the sensitivity matrix A.

We now wish to devise an operation to be applied to Z which will yield an estimate of X by minimizing a certain loss function. In the following let X be the true parameters and $\hat{X}$ be the estimate of X; therefore, the parameter estimation error is defined by $\tilde{X} = X - \hat{X}$.

A. Bayesian and Weighted Least Squares Estimation

A Bayesian or weighted least squares estimation algorithm can be developed[9] for Eq. (3). If $f(\cdot)$ is linear in X, the solution is obtained in one step. If $f(\cdot)$ is nonlinear in X, the parameters can be estimated using the linearized measurement equation and iterated with the algorithm given below.

A formulation for Bayesian estimation, which accounts for statistical information about the parameters X and the measurements Z can be obtained by minimizing the objective function in Eq. (7) with respect to the parameters to be estimated, $\hat{X}$. For weighted least squares estimation the inverse of the measurement noise and parameter covariance matrices, R^{-1} and P^{-1} are simply weighting matrices. The objective function, Eq. (7), consists of a part that minimizes the error between the measurements and predicted measurements, and a part that minimizes the parameter change from the current or nominal value:

$$\min_{\delta\hat{X}} (\delta\hat{X} - \delta X^0)^T P_0^{-1}(\delta\hat{X} - \delta X^0) + (\delta Z - A\delta\hat{X})^T R^{-1}(\delta Z - A\delta X) \tag{7}$$

where

$$\delta\hat{X} = \hat{X} - X^n \tag{8}$$

$$\delta X^0 = X^0 - X^n \tag{9}$$

The function can be minimized by taking partial derivatives with respect to $\delta\hat{X}$ and equating the resulting expression to zero. By using the matrix inversion lemma,[18] and assuming zero mean noize, $E[V] = 0$, the result can be expressed in the form

$$\delta\hat{X} = \delta\hat{X}^0 + P_{\tilde{X}}A^T\boldsymbol{R}^{-1}(\delta Z - A\delta\hat{X}^0) \tag{10}$$

In Eq. (10), $P_{\tilde{X}}$ is the covariance matrix of the parameter estimate error which can be shown to be[19,20]

$$P_{\tilde{X}} = E[X - \hat{X}][X - \hat{X}]^T = [P_0^{-1} + A^T R^{-1} A]^{-1} \tag{11}$$

It can also be shown that $P_{\delta\tilde{X}} = P_{\tilde{X}}$ and $P_{\delta X^0} + P_{X^0} \stackrel{\text{def}}{=} P_0$. Expanding the terms in Eq. (10) we obtain,

$$\hat{X} - X^n = \hat{X}^0 - X^n + P_{\tilde{X}}A^T R^{-1}(Z - f(X^n) - A(\hat{X}^0 - X^n)) \tag{12}$$

Rearranging terms we get the desired expression:

$$\hat{X} = \hat{X}^0 + P_{\tilde{X}}A^T R^{-1}(Z - f(X^n) - A(\hat{X}^0 - X^n)) \tag{13}$$

Anticipating an iterative process for this batch estimation procedure, these equations can be rewritten as

$$\hat{X}_{i+1} = \hat{X}^0 + P_{i+1}A_{i+1}^T R^{-1}(Z - f(X^n) - A_{i+1}(\hat{X}^0 - X_i^n)) \tag{14}$$

$$P_{i+1} = [P_0^{-1} + A_{i+1}^T R^{-1} A_{i+1}]^{-1} \tag{15}$$

where

$$A_i = \left[\frac{\partial f(X)}{\partial X}\right]_{X_i^n} \tag{16}$$

X_i^n is the state about which we linearize $f(X)$ for the ith iteration and

$$P_{\tilde{X}i+1} \stackrel{\text{def}}{=} P_{i+1} \tag{17}$$

for notational simplicity. The obvious choice for X_i^n rb is $\hat{X}_i$. During the iteration procedure, only the last P_{i+1} should be interpreted as the error covariance matrix. The intermediate values for P_{i+1} do not have statistical significance.

We also note that the above iterative procedure is equivalent to defining a sequence of performance indices, where at the $i + 1$th iteration we minimize the following:

$$\min_{\hat{X}_{i+1}} (\hat{X}^0 - \hat{X}_{i+1})^T P_0^{-1}(\hat{X}^0 - \hat{X}_{i+1}) + (Z - f(\hat{X}_{i+1}))^T R^{-1}(Z - f(\hat{X}_{i+1})) \tag{18}$$

B. Mathematical Programming

The mathematical programming technique of system identification uses optimization theory to find the *optimum* set of design variables. The optimum set of design variables is calculated by minimizing an objective function. An objective function that can have a wide range of application is shown in Eq. (19). It is motivated by the objective function in Eqs. (7) and (10) which have: 1) the error between the measurements and the modal predictions, and 2) the change in the model parameters.

Weighting factors are used to weight the two parts of the objective function, as well as the individual measurements and parameters. The mathematical programming techniques also have the ability to incorporate *side limits* on the parameters, which are upper and lower bounds on the parameters. Equality and inequality constraints that are nonlinear functions of the measurements and parameters can also be imposed using this class of techniques.

$$F = W_1(\{Z - \hat{Z}\}^T[W_Z]\{Z - \hat{Z}\}) + W_2(\{X - X_0\}^T[W_X]\{X - X_0\}) \tag{19}$$

IV. Truss Structure Application

Figure 1 shows the truss structure on which a structural control experiment was successfully implemented.[21] Figure 2 is a schematic of the truss structure which shows locations of sensors and actuators, and the control performance objectives. The truss structure is made of polycarbonate 1-in. diameter tubing. The truss has 1-ft cubic bays, which are assembled with the tubes bonded to polycarbonate corner blocks. The truss is mounted on a 2000 lb inertial mass, which is supported on air bags. Finite element modeling of the truss is straightforward, since the geometry and load paths are well defined. The structural control system used piezoelectric sensors and actuators attached to diagonal struts of the bottom two bays, which sense and actuate strain. The truss finite element model was to be used as a basis for designing a structural control system.

The initial NASTRAN model of the truss had modal frequency errors in the first 10 elastic modes exceeding 15% (Table 1). Using a weighted least squares system identification technique to estimate Young's modulus (E), the modal frequency error was reduced to approximately 5%. The value of Young's modulus was reduced from the nominal value which probably compensated for joint flexibility in the truss. The updated model would probably be quite adequate for response analysis of a structure; however, the mode shapes were slightly in error as shown in Figs. 3 and 4. The first four elastic modes occurred in two closely spaced pairs with highly coupled dynamics.

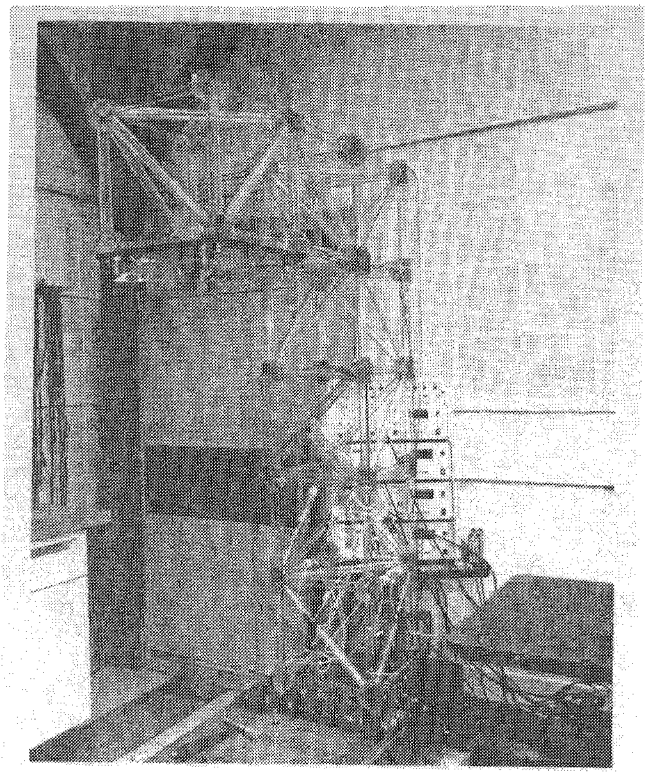


Fig. 1 The truss structure.

The control system designed using the structural model with an updated E was unstable at approximately 10 Hz. Since the *bounce modes* of the inertial mass on the air bags were at approximately 2 Hz, it was suspected that the strain in the lower truss bays were influenced by the boundary stiffness (i.e., air bags), which was not modeled. Vertical springs modeling the air bags were included in the model, and the weighted least squares technique was used to simultaneously estimate the air bag stiffness (K) as well as E.

Table 1 shows the resulting natural frequencies of the identified system. The control design based on this model was stable.[21] Figures 3 and 4 show that the 10-Hz modes from the test and the NASTRAN model with updated E and K compare very well. There is also excellent mode shape correlation for all of the first 10 elastic modes.

V. Electronics Package Application

A structural model of an electronics package shown in Fig. 5 was needed for inclusion in a larger system model in which system level response calculations would be performed. The model was needed for development of shock and vibration component test specifications. Several finite element models were created for this structure, including an 8000 degree-of-freedom (DOF) 3-D solid model, a 200 DOF 3-D beam model, and a simplified 31 DOF 2-D beam and lumped parameter model. None of the models satisfactorily matched the test data, and the 31 DOF model was selected for the parameter estimation study. This model represented a crude but physical mesh of the hardware. A schematic is shown in Fig. 6.

The electronics package weighs approximately 17 lb and consists of a metal housing containing a number of electronic components which are imbedded in a stiff foam. The internal component stack is physically attached to the upper and lower portions of the housing by a rigid epoxy bond. The components themselves are internally potted microcircuits and printed

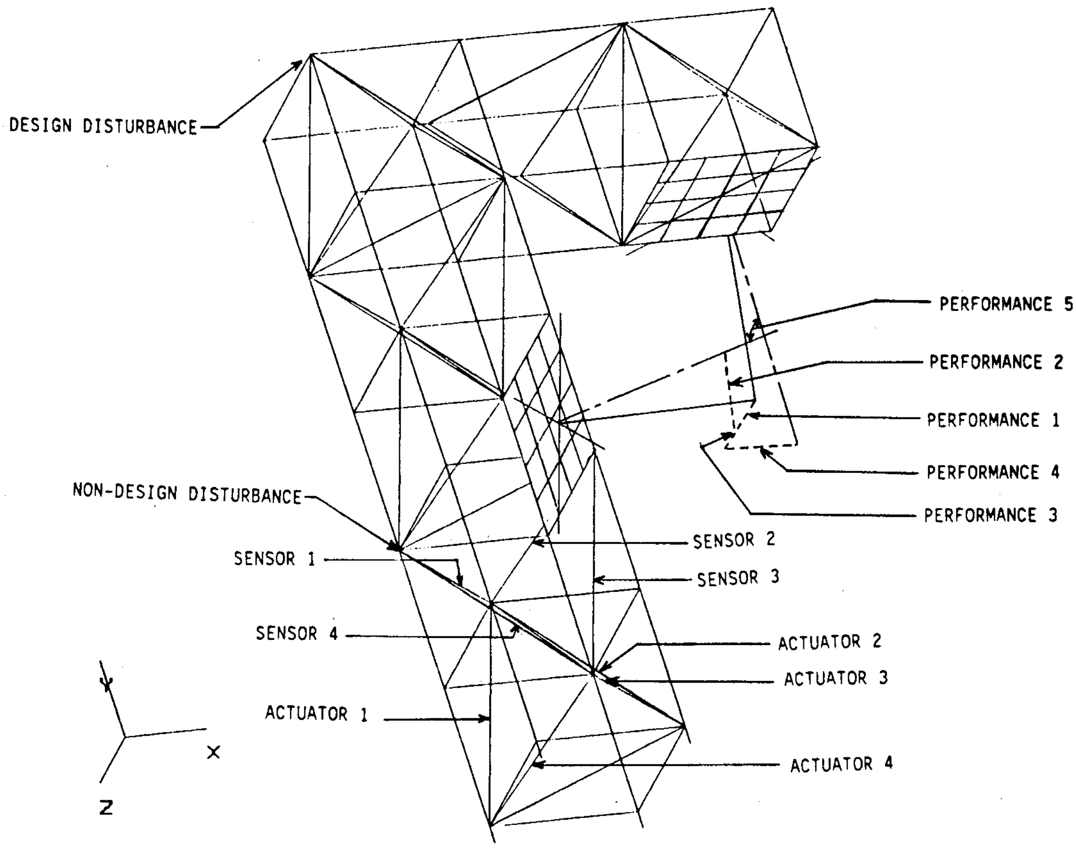
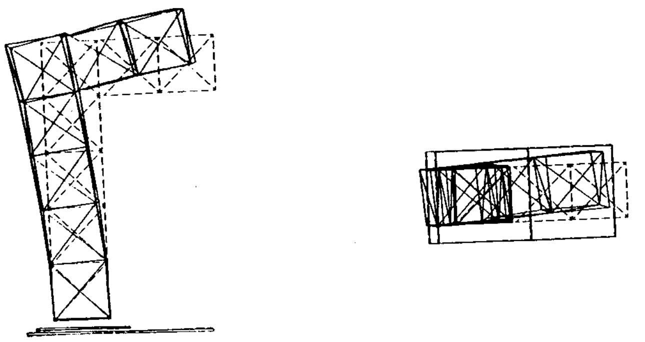


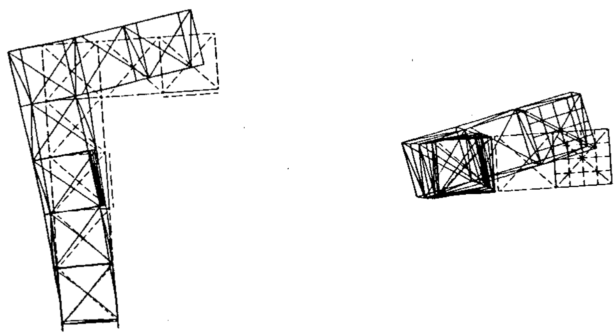
Fig. 2 Truss schematic.

Table 1 Truss weighted least squares parameter estimation results

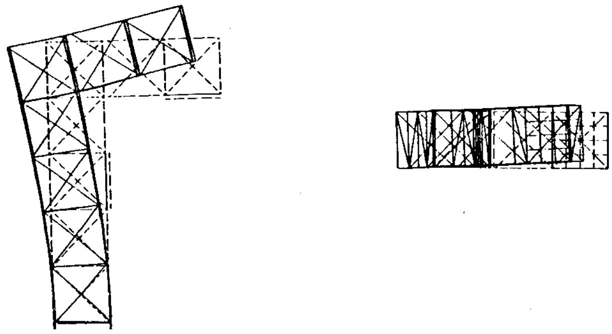
Mode	Modal test frequencies, Hz	Initial NASTRAN model Freq., Hz	Initial NASTRAN model Error, %	NASTRAN with updated E Freq., Hz	NASTRAN with updated E Error, %	NASTRAN with updated E & K Freq., Hz	NASTRAN with updated E & K Error, %
1	10.28	11.12	8.04	10.25	0.30	10.28	0.0
2	10.84	11.41	5.24	10.46	3.51	10.87	0.28
3	19.86	21.37	7.56	19.30	2.80	19.32	2.72
4	21.28	24.03	12.50	21.65	1.64	21.34	0.28
5	45.70	51.18	12.00	46.28	1.23	45.87	0.37
6	60.45	65.80	8.46	59.25	1.98	59.32	1.87
7	62.46	71.30	13.50	64.15	2.58	63.04	0.93
8	78.32	87.86	12.18	79.18	1.10	79.25	1.19
9	88.70	97.14	9.52	87.69	1.14	86.90	2.04
10	115.47	134.17	16.19	120.69	4.52	119.29	3.30



a) Experimental (10.28 Hz)

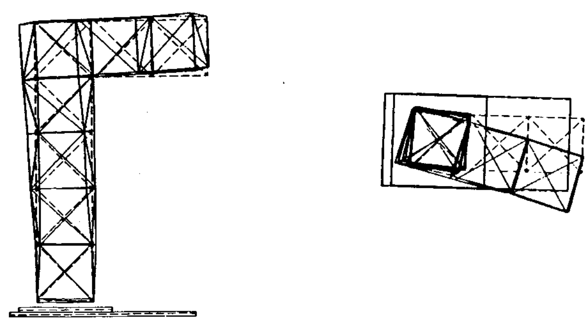


b) Structural model with updated *E* (10.25 Hz)

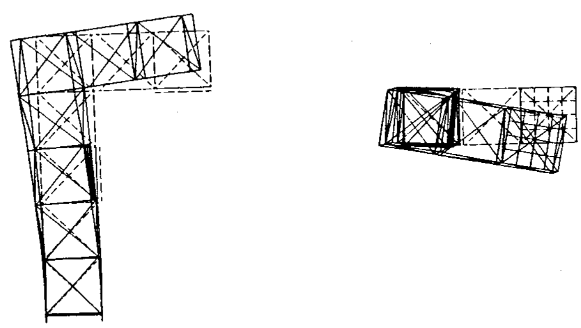


c) Structural model with updated *E* and *K* (10.28 Hz)

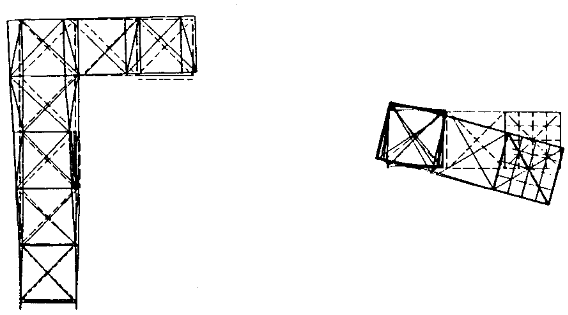
Fig. 3 First elastic truss experimental and analytical mode shapes.



a) Experimental (10.84 Hz)



b) Structural model with updated *E* (10.46 Hz)



c) Structural elastic truss experimental and analytical mode shapes.

Fig. 4 Second elastic truss experimental and analytical mode shapes.

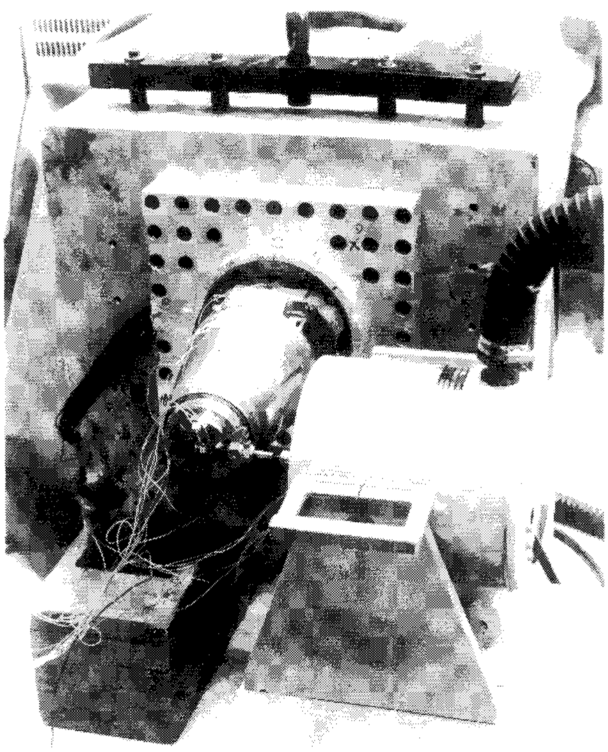


Fig. 5 Electronics package.

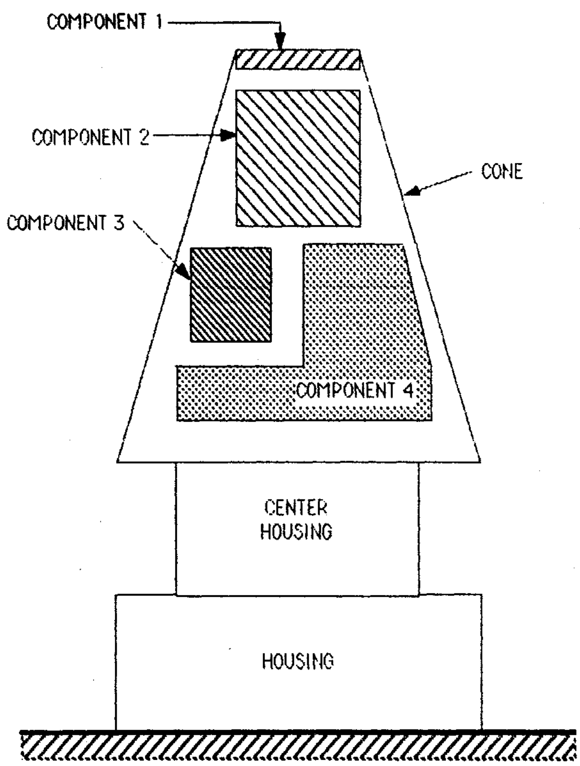


Fig. 6 Electronics package schematic.

wiring boards with light metal housings containing the potted components. This type of structure is very difficult to model due to the large ratio of nonstructural to structural mass, its many nonmetallic structural load paths, and the flexibility of the individual components.

A modal test of the package was performed to characterize its dynamic response and to acquire data for the model verification.[22] The test unit was instrumented internally on the five major internal components and externally on the housing. In both the finite element analysis model and the modal test, the internal components were assumed to behave as rigid bodies. Both frequency response data and modal frequency/mode shape information were acquired in the tests.

The system identification results reported in this paper were obtained using mathematical programming techniques with modal frequency data. Earlier studies using Bayesian estimation with frequency response data showed that it was mandatory to estimate both mass and stiffness terms. The one-step system identification methods were also applied to this model using both mode shape and modal frequency data. Results from these related studies are not reported here.

The objective function used in the mathematical programming study chose the weights in Eq. (19), such that zero penalty was imposed for parameter changes and the modal frequency error in the first eight elastic modes was minimized. The mass and stiffness parameters used in the 31 DOF structural model could possibly have significant errors due to the techniques that were necessary to develop the low-order model. The modal frequency percentage error was selected to provide an equitable weighting for the low- and high-frequency modes.

The initial model updating attempt used 17 parameters which included all of the major stiffnesses in the foamed internal portion of the package. The second and fourth modes did not improve as much as the other modes included in the objective function. The major differences in the mode shapes appeared to be located in the vicinity of components 1, 2, and 3, which are near the top of the electronics package. A subsequent estimation was performed using 19 parameters which included additional stiffness parameters between component 2 and components 3 and 4. The 19-parameter estimation run used initial parameter values equal to the final parameter values from the 17-parameter run. The results shown in Table 2 did not appreciably affect the two modes intended; however, the modal frequency error in the highest three modes was reduced. The mass and inertia of component 3 were then included as parameters in the estimation. The new 21-parameter estimation used initial parameter values equal to the final parameter values from the 19-parameter run. The result of

Table 2 Electronics package mathematical programming parameter estimation results

				Parameter estimation results					
	Test data	Initial model		17 Parameters		19 Parameters		21 Parameters	
Mode	Frequencies, Hz	Freq., Hz	Error, %	Freq., Hz	Error, %	Freq., Hz	Error, %	Freq., Hz	Error, %
1	296.0	190.8	35.5	295.6	0.0	293.5	0.9	295.6	0.0
2	544.0	387.4	28.8	457.3	15.9	460.8	15.3	498.3	8.4
3	614.0	567.9	7.5	602.4	1.9	597.9	2.6	622.3	1.3
4	660.0	785.1	19.0	721.2	9.3	718.9	8.9	701.1	6.2
5	884.0	939.7	6.3	909.5	2.9	898.9	1.7	923.2	4.4
6	1203.0	1098.5	8.7	1220.0	1.4	1200.9	0.2	1215.8	1.1
7	1571.0	1534.3	2.3	1624.2	3.4	1582.2	0.7	1585.6	0.9
8	1864.0	1943.5	4.3	1992.4	6.9	1871.2	0.4	1919.8	3.0

the 21-parameter estimation is shown in Table 2. The modal frequency error of the second and fourth modes is reduced to an acceptable level (i.e., within measurement errors and hardware differences).

The 21-parameter estimation problem was restarted from the original parameter values. These results were very similar to those obtained using the sequence of runs with different numbers of parameters. Figure 7 shows the convergence of the objective function for the 21-parameter estimation starting with the original parameters, as well as the sequence of estimations with 17, 19, and 21 parameters. The objective function decreases to the same value for both cases. The first eight elastic mode shapes for the test and updated analytical model compared well. Figure 8 shows two of these modes. Figure 9 compares the measured frequency response function at component 1 with the original and updated analytical models.

In summary, the new stiffness parameters were changed within acceptable limits. The mass and inertia of component 2, which is a small component relative to the overall electronic package mass, was changed by amounts consistent with the foam that was arbitrarily lumped at that position. The overall mass change to the electronics package also was small and within unit-to-unit variations. Only modal frequency data were used in the identification algorithm. However, both mode shapes and frequency response functions were evaluated.

VI. Conclusions

A system was developed that integrated commercial software for finite element modeling, mathematical programming techniques, and general linear system analysis. The use of this integrated system identification software was illustrated with two very different applications: estimation of the material

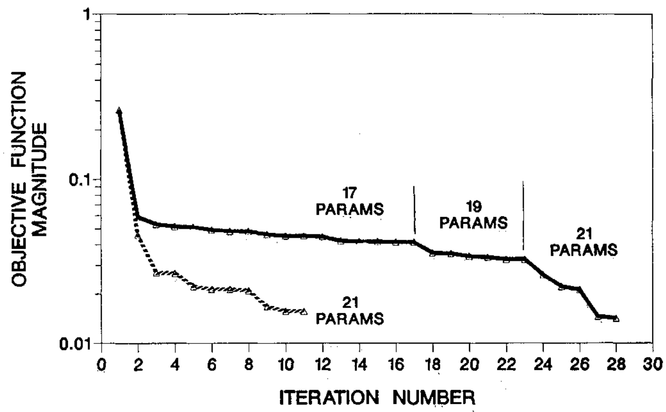


Fig. 7 Electronics package objective function convergence.

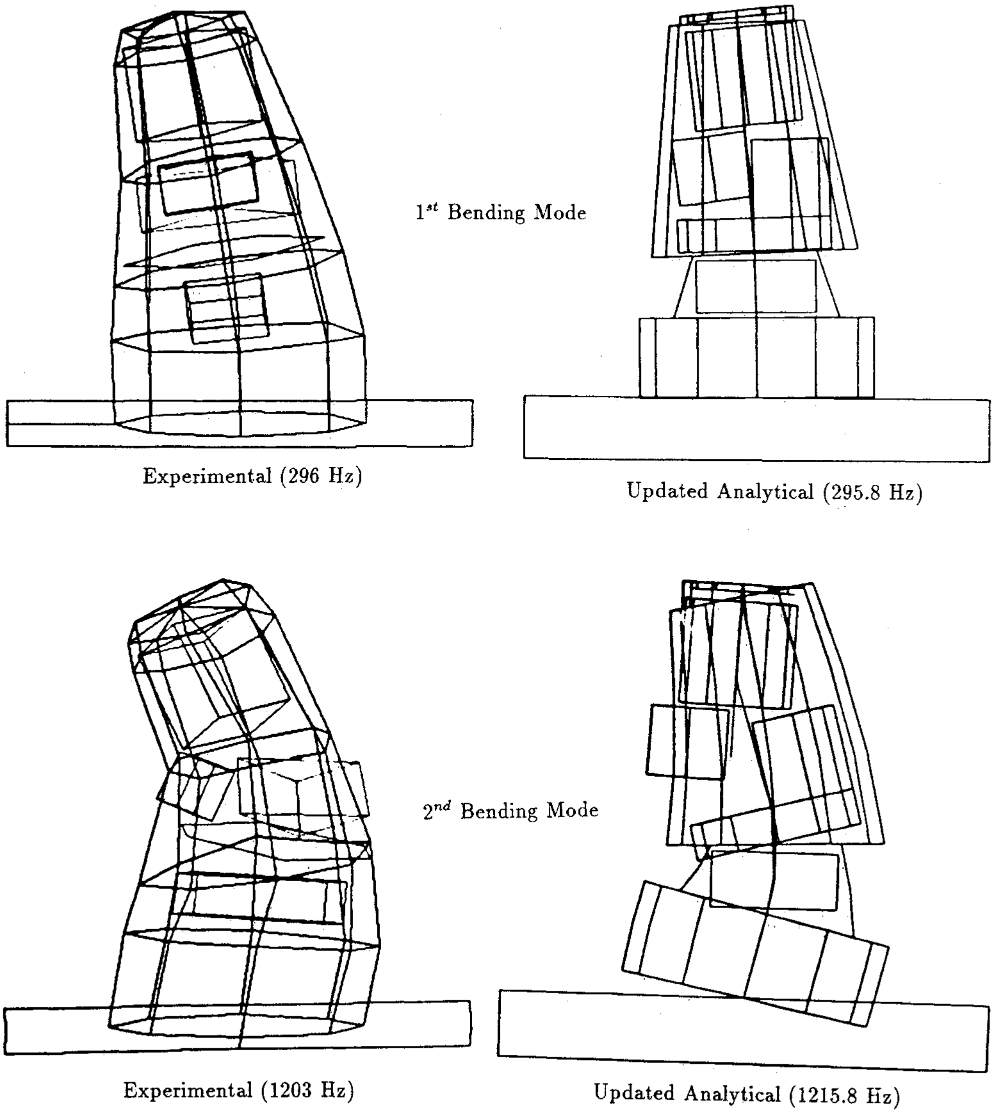


Fig. 8 Electronics package mode shape comparison.

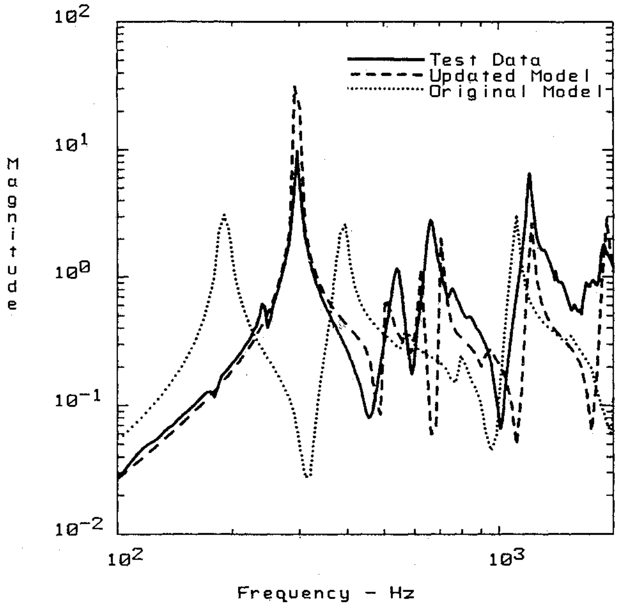


Fig. 9 Electronics package frequency response function at component 1.

constants of a truss structure, and estimation of the stiffness and mass properties of an electronics package. In both cases, experimental modal test data were used in the estimation procedures.

The truss structure was easily modeled and had well-defined geometry and load paths. Parameter estimation was applied to a limited number of uncertain crucial parameters yielding a model that gave excellent results when used in a control design experiment. The electronics package was very difficult to model with finite elements because of significant nonstructural mass and ill-defined internal load paths. There were many uncertain parameters to be estimated. The initial task in the updating of the electronics package model was to find a set of parameters applicable to the identification problem. System identification was successfully used to produce an improved model acceptable for response analysis.

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